In the same connection the following formula will be found of use :—

$$\int uv\,dx \log \int uv\,dx \leq \int uv \log \frac{v}{u^{a-1}}\,dx + \int uv\,dx \log \int u^a\,dx.$$

Such inequalities will be found useful, for example, in extending the results on Fourier series contained in my paper on "Successions of Integral and Fourier series."

§ 7. We thus obtain, for example, theorems of the following type :—

If S_n *denote the n-th Cesàro partial summation of the Fourier series of a positive function* $f(x)$, *which is such that* $f(x)\log f(x)$ *is summable, and* T_n *denote the n-th Cesàro partial summation of the Fourier series of the function* $f(x)\log f(x)$, *then*

$$S_n(x) \log S_n(x) \leq T_n(x).$$

From this also we deduce that, *if* $f_n(x)$ *denote the n-th Cesàro partial summation of the Fourier series of a function* $f(x)$ *which is such that* $|f(x)|\log|f(x)|$ *is summable, then* $\int_E |f_n(x)|\log|f_n(x)|\,dx$ *has the unique double limit zero, when* $E \to 0$ *and* $n \to \infty$; *and that*

$$\operatorname*{Lt}_{n\to\infty} \int_c^z \left| f_n(x) \right| \log \left| f_n(x) \right| dx = \int_c^z \left| f(x) \right| \log \left| f(x) \right| dx.$$

Finally we may remark that from analogous considerations we may establish the truth of the following general theorem on double successions :—

If $f_n(x)$ *and* $g_m(x)$ *denote the Cesàro summations of the first* $(2n+1)$ *terms of the Fourier series of two functions* $f(x)$ *and* $g(x)$, *belonging to a pair of complementary classes of summable functions, such as those here considered, then*

$$\operatorname*{Lt}_{m\to\infty,\, n\to\infty} \int f_n(x)\, g_m(x)\,dx = \int f(x)\, g(x)\,dx.$$

Negative After-Images and Successive Contrast with Pure Spectral Colours.

By A. W. Porter, B.Sc., F.R.S., and F. W. Edridge-Green, M.D., F.R.C.S.

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